Name ______________________ Student Number ______________

All solutions are to be presented on the paper in the space provided. The quiz is open book. You can discuss the problem with others and ask the TA questions.

(1) Find the following limits:

(a) $\lim_{x\to\infty} \frac{x-1}{|x-1|}$

$$= \lim_{x\to\infty} \frac{x-1}{x-1}, \qquad \text{since } |x-1| = x-1 \text{ when } x \geq 1$$
$$= \lim_{x\to\infty} 1$$
$$= 1$$

(b) $\lim_{x\to-\infty} \frac{2x}{\sqrt{x^2-1}}$

$$= \lim_{x\to-\infty} \frac{2x}{\sqrt{x^2(1-\frac{1}{x^2})}}$$
$$= \lim_{x\to-\infty} \frac{2x}{\sqrt{x^2}\sqrt{1-\frac{1}{x^2}}}$$
$$= \lim_{x\to-\infty} \frac{2x}{|x|\sqrt{1-\frac{1}{x^2}}}$$
$$= \lim_{x\to-\infty} \frac{2x}{-x\sqrt{1-\frac{1}{x^2}}} \qquad \text{since } |x| = -x \text{ when } x < 0$$
$$= \lim_{x\to-\infty} \frac{2}{-\sqrt{1-\frac{1}{x^2}}}$$
$$= -2$$

(2) Use the intermediate value theorem to show that the equation $\sin x = -e^x$ has a solution by answering the following questions:

(a) Convert the equation to a function, $f(x)$, and show that $f(x)$ is continuous.
$f(x) = \sin(x) + e^x$ is continuous since it is the sum of continuous functions.

(b) Find an interval $[a, b]$ for which $f(a) < 0$ and $f(b) > 0$.
Guess. Use numbers that are easy to compute with.
Let $b = 0$ and $a = -\frac{\pi}{2}$. If you have a graphing calculator, plot $f(x)$ and see where it crosses the x axis. Pick an a to the left of this point and a b to the right *as long as* $f(x)$ *does not cross the* x *axis more than once in* $[a, b]$.

$$f(0) = 0 + 1 = 1 > 0$$
$$f(-\frac{\pi}{2}) = -1 + e^{-\frac{\pi}{2}} < 1$$

You're not going to be able to compute a number for $f(-\frac{\pi}{2})$, so you need to note that

$$e^{-\frac{\pi}{2}} = \frac{1}{e^{\frac{\pi}{2}}} < 1 \text{ since } e^{\frac{\pi}{2}} > 1.$$

This requires that you know something about e, namely that e is about 2.7.

(c) Clearly state how the above information and the intermediate value theorem guarantee a solution of $\sin x = -e^x$ in the interval (a, b).
Since $f(x)$ is continous, and 0 is between $f(-\frac{\pi}{2})$ and $f(0)$, the intermediate value theorem guarantees that there is a number c between $-\frac{\pi}{2}$ and 0 such that $f(c) = 0$. When $f(c) = 0$, we get $\sin c + e^c = 0$, or $\sin c = -e^c$, so c solves the original equation.